

Deformed Super-Yang-Mills in Batalin-Vilkovisky Formalism

Mir Faizal
Mathematical Institute
University of Oxford
Oxford OX1 3LB

September 12, 2012

Abstract

In this paper we will analyse a three dimensional super-Yang-Mills theory on a deformed superspace with boundaries. We show that it is possible to obtain an undeformed theory on the boundary if the bulk superspace is deformed by imposing a non-vanishing commutator between bosonic and fermionic coordinates. We will also analyse this theory in the Batalin-Vilkovisky (BV) formalism and show that these results also hold at a quantum level.

Key Words: Noncommutative superspace, Batalin-Vilkovisky formalism.

1 Introduction

The supersymmetric transformation of $\mathcal{N} = 1$ super-Yang-Mills theory in three dimensions is a total derivative, and thus for manifolds with boundaries it generates a boundary term [1]-[3]. This effectively breaks the supersymmetry of the theory. However, this problem can be resolved by imposing boundary conditions. Alternately, the action can be made supersymmetric even before imposing boundary conditions. This is done by the addition of a suitable boundary term which exactly cancels the term generated by the supersymmetric transformation of the bulk Lagrangian. Supersymmetry can

also be broken by imposing anti-commutativity between the Grassman coordinates of the superspace. Such deformation of superspace occur due to RR fields [4]-[5]. The deformations of a $\mathcal{N} = 4$ super-Yang-Mills theory can also occur due to a graviphoton backgrounds [6]-[7]. Supersymmetric Yang-Mills theory with chiral matter multiplets in presence of graviphoton background has also been analysed [8]. In fact, a perturbation theory scheme has been used for computing correlation functions for this theory.

As the Lagrangian for the super-Yang-Mills theory is invariant under gauge transformations, so we have to add a gauge fixing term and a ghost term before doing any calculations. The new Lagrangian thus obtained is invariant under a new set of symmetries called the BRST and the anti-BRST symmetries. The BRST and the anti-BRST symmetries for the ordinary Yang-Mills theory has already been investigated [9]-[12]. The BRST symmetry of a $\mathcal{N} = 1$ super-Yang-Mills theory has also been studied in superspace formalism [13]-[14]. This theory was analysed in the background field method. In this method all the fields in a theory are shifted. The BRST and the anti-BRST symmetries of these shifted fields is then analysed by using Batalin-Vilkovisky formalism [15]-[21]. In this formalism the extended BRST and the extended anti-BRST symmetries arise due to the invariance of a theory under both the original BRST and the original anti-BRST transformations along with the shift transformation. This have been done for the conventional Yang-Mills theories [22]-[28]. Furthermore, the BRST and the anti-BRST symmetries can be viewed as supersymmetric transformations because they mix the fermionic and bosonic coordinates. Thus, the extended BRST and the extended anti-BRST symmetries for Yang-Mills theory have been studied in the extended superspace formalism [29]-[30].

In this paper we will analyse the super-Yang-Mills theory in presence of boundary. We will show that even if we deform the superspace of this original theory, it is possible to obtain a undeformed theory on the boundary. This is because the boundary only preserves half the supersymmetry of the bulk theory. Hence, if we perform a deformation in the other half, it will not be visible on the boundary. We will also analyse this theory in the BV-formalism and hence show that this result holds even at a quantum level.

2 Deformed Super-Yang-Mills Theory

In this section we shall first review the properties of the super-covariant derivatives for non-Abelian $\mathcal{N} = 1$ gauge fields in three dimensions. In order to do that we first introduce a two component anti-commuting Grassmann parameters θ_a . We also introduce two anti-symmetric tensors C^{ab} and C_{ab} which are used to raise and lower spinor indices. These anti-symmetric tensors satisfy $C_{ab}C^{bc} = \delta_a^c$. Furthermore, we let $\theta^2 = \theta_a C^{ab} \theta_b / 2 = \theta^a \theta_a / 2$. Now Γ_a is defined to be a matrix valued spinor superfield suitable contracted with generators of this Lie algebra, $\Gamma_a = \Gamma_a^A T_A$, where T_A are Hermitian generators of a Lie algebra $[T_A, T_B] = if_{AB}^C T_C$. We also define super-covariant derivatives as

$$\nabla_a = D_a - i\Gamma_a, \quad (1)$$

where D_a is the super-derivative given by

$$D_a = \partial_a + (\gamma^\mu \partial_\mu)_a^b \theta_b. \quad (2)$$

The matrix valued spinor superfield transforms under gauge transformations as

$$\delta\Gamma_a = \nabla_a \Lambda. \quad (3)$$

The components of the superfield Γ_a are given by

$$\begin{aligned} \chi_a &= [\Gamma_a]_1, & A &= -\frac{1}{2}[\nabla^a \Gamma_a]_1, \\ A^\mu &= -\frac{1}{2}[\nabla^a (\gamma^\mu)_a^b \Gamma_b]_1, & E_a &= -[\nabla^b \nabla_a \Gamma_b]_1. \end{aligned} \quad (4)$$

Now let \mathcal{D}_μ be the conventional covariant derivative given by

$$\mathcal{D}_\mu = \partial_\mu - iA_\mu, \quad (5)$$

then it can be shown by direct computation that the super-covariant derivative satisfies

$$\{\nabla_a, \nabla_b\} = 2\gamma_{ab}^\mu \mathcal{D}_\mu. \quad (6)$$

For a $\mathcal{N} = 1$ superfields in three dimensions $[\nabla_a, \nabla_b]$ must be proportional to the anti-symmetric tensor C_{ab} [31]-[32]. Thus, we can write

$$\begin{aligned} \nabla_a \nabla_b &= \frac{1}{2}\{\nabla_a, \nabla_b\} + \frac{1}{2}[\nabla_a, \nabla_b] \\ &= \gamma_{ab}^\mu \mathcal{D}_\mu - C_{ab} \nabla^2. \end{aligned} \quad (7)$$

Furthermore, using [1],

$$D_a D_b D_c = \frac{1}{2} D_a \{D_b, D_c\} - D_b \{D_a, D_c\} + \frac{1}{2} D_c \{D_a, D_b\}, \quad (8)$$

we find that the super-covariant derivative satisfies

$$\nabla_a \nabla_b \nabla_c = \frac{1}{2} \nabla_a \{\nabla_b, \nabla_c\} - \nabla_b \{\nabla_a, \nabla_c\} + \frac{1}{2} \nabla_c \{\nabla_a, \nabla_b\}. \quad (9)$$

Thus, we get

$$\nabla^a \nabla_b \nabla_a = 0, \quad (10)$$

$$\nabla^2 \nabla_a = (\gamma^\mu \nabla)_a \mathcal{D}_\mu, \quad (11)$$

Now the non-abelian Yang-Mills theory on this superspace can now be written as [35],

$$\mathcal{L}_{bulk} = \nabla^2 Tr(\omega^a \omega_a)| \quad (12)$$

where

$$\omega_a = \frac{1}{2} D^b D_a \Gamma_b - \frac{i}{2} [\Gamma^b, D_b \Gamma_a] - \frac{1}{6} [\Gamma^b, \{\Gamma_b, \Gamma_a\}]. \quad (13)$$

and $'|'$ means that the quantity is evaluated at $\theta_a = 0$. This Lagrangian in component form is given by

$$\mathcal{L}_{bulk} = 4F^{\mu\nu} F_{\mu\nu} + 2\bar{\lambda} \gamma^\mu \mathcal{D}_\mu \lambda. \quad (14)$$

It may be noted that a theory with $\mathcal{N} = 1$ supersymmetry in four dimensions has the same amount of supersymmetry as a theory with $\mathcal{N} = 2$ supersymmetry in three dimensions. In four dimensions two independent super-charges exist, so there are two terms like this. Similarly, a Yang-Mills theory with $\mathcal{N} = 2$ supersymmetry in three dimensions will have two terms like this. This Lagrangian is invariant under supersymmetric transformations generated by the supercharge Q_a given by

$$Q_a = \partial_a - (\gamma^m \partial_m)_a^b \theta_b. \quad (15)$$

It satisfies the following algebra,

$$\{Q_a, Q_b\} = 2(\gamma^\mu)_{ab} \partial_\mu. \quad (16)$$

However, in presence of a boundary half of this supersymmetry is broken. To understand how this works let us put a boundary at fixed x^3 . Now μ splits into $\mu = (m, 3)$, here m labels the coordinates along the boundary. The induced values of the super-derivative D_a and the super-covariant derivative ∇_a on the boundary are denoted by D'_a and ∇'_a , respectively. This boundary super-derivative D'_a is obtained by neglecting $\gamma^3 \partial_3$ contributions in D_a ,

$$D'_a = \partial_a + (\gamma^m \partial_m)_a^b \theta_b. \quad (17)$$

The boundary super-covariant derivative ∇'_a can thus be written using Γ'_a which is the induced values of the bulk field Γ_a on the boundary. Any boundary field along with the induced value of any quantity e.g., Λ on the boundary will be denoted by Λ' . This convention will be followed even for component fields of superfields. The matrix valued spinor superfield Γ'_a transforms under gauge transformations as

$$\delta \Gamma'_a = \nabla'_a \Lambda'. \quad (18)$$

Now we define a projection operator P_\pm as

$$(P_\pm)_a^b = \frac{1}{2}(\delta_a^b \pm (\gamma^3)_a^b). \quad (19)$$

The projected values of the super-covariant derivative satisfy [31]-[32]

$$\nabla_{+a} \nabla_{+b} = -(\gamma^+)_{ab} \mathcal{D}_+, \quad (20)$$

$$\nabla_{-a} \nabla_{-b} = -(\gamma^-)_{ab} \mathcal{D}_-, \quad (21)$$

$$\nabla_{-a} \nabla_{+b} = -(P_-)_{ab} (\mathcal{D}_3 + \nabla^2), \quad (22)$$

$$\nabla_{+a} \nabla_{-b} = (P_+)_{ab} (\mathcal{D}_3 - \nabla^2). \quad (23)$$

Furthermore, we can obtain the following algebra from these relations,

$$\{\nabla_{+a}, \nabla_{+b}\} = -2(\gamma^+)_{ab} \mathcal{D}_+, \quad (24)$$

$$\{\nabla_{-a}, \nabla_{-b}\} = -2(\gamma^-)_{ab} \mathcal{D}_-, \quad (25)$$

$$\{\nabla_{-a}, \nabla_{+b}\} = -2(P_-)_{ab} \mathcal{D}_3. \quad (26)$$

The generators of the supersymmetry also satisfy a similar algebra

$$\{Q_{+a}, Q_{+b}\} = 2(\gamma^+)_{ab} \partial_+, \quad (27)$$

$$\{Q_{-a}, Q_{-b}\} = 2(\gamma^-)_{ab} \partial_-, \quad (28)$$

$$\{Q_{-a}, Q_{+b}\} = 2(P_-)_{ab} \partial_3. \quad (29)$$

It is possible to add a boundary term to the super-Yang-Mills theory to make it preserve half the supersymmetry, even in presence of a boundary [1]. This is because we can write the generators of the supersymmetry as

$$\epsilon^a Q_a = \epsilon^+ Q_+ + \epsilon^- Q_- . \quad (30)$$

Now it is possible to break the supersymmetry corresponding to Q_- and retain the supersymmetry corresponding to Q_+ . This is done by addition the following term to the bulk Lagrangian

$$\mathcal{L}_{boud} = \mathcal{D}_3 Tr(\omega^a \omega_a)|. \quad (31)$$

This term exactly cancels the boundary term generated by the supersymmetric variation of the bulk Lagrangian. Now we have

$$\begin{aligned} -\nabla_+ \nabla_- &= -C^{ab} \nabla_{+a} \nabla_{-b} \\ &= -C^{ab} (P_+)_{ab} (\mathcal{D}_3 - \nabla^2) \\ &= -(P_+)^a_a (\mathcal{D}_3 - \nabla^2) \\ &= (\mathcal{D}_3 - \nabla^2). \end{aligned} \quad (32)$$

So, we can write the Lagrangian for supersymmetric Yang-Mills theory in presence of a boundary as

$$\begin{aligned} \mathcal{L}_{bb} &= \mathcal{L}_{bulk} + \mathcal{L}_{boud} \\ &= (\mathcal{D}_3 - \nabla^2) Tr(\omega^a \omega_a)| \\ &= \nabla_+ \nabla_- Tr(\omega^a \omega_a)|. \end{aligned} \quad (33)$$

This Lagrangian in component form is given by

$$\mathcal{L}_c = 4F^{\mu\nu} F_{\mu\nu} + 2\bar{\lambda} \gamma^\mu \mathcal{D}_\mu \lambda + \mathcal{D}_3(\bar{\lambda} \lambda). \quad (34)$$

Now we shall deform this three dimensional Yang-Mills theory. To do so we first let $y^\mu = x^\mu + \theta^a (\gamma^\mu)_a^b \theta_b$. Then we promote the superspace coordinates to operators $\hat{\theta}^a$ and \hat{y}^μ and impose the following deformation of the superspace algebra on them,

$$[\hat{y}^\mu, P_- \hat{\theta}^a] = A^{\mu a}. \quad (35)$$

This deformation occurs due to a graviphoton background [6]-[8]. Unlike a RR deformation, this deformation does not break any supersymmetry of the

theory. A matrix valued superfields $\hat{\Gamma}_a(\hat{y}, \hat{\theta})$ on this deformed superspace is obtained by suitably contracting it with generators of the Lie algebra $[T_A, T_B] = if_{AB}^C T_C$, in the adjoint representation,

$$\hat{\Gamma}_a(\hat{y}, \hat{\theta}) = \hat{\Gamma}_a^A(\hat{y}, \hat{\theta}) T_A. \quad (36)$$

We now use Weyl ordering and express the Fourier transformation of $\hat{\Gamma}_a(\hat{y}, \hat{\theta})$ as [33]-[34],

$$\hat{\Gamma}_a(\hat{y}, \hat{\theta}) = \int d^3 k \int d^2 \pi e^{-ik\hat{y} - \pi\hat{\theta}} \Gamma_a(k, \pi). \quad (37)$$

Now we can also express Fourier transformation of $\Gamma_a(y, \theta)$ as,

$$\Gamma_a(y, \theta) = \int d^3 k \int d^2 \pi e^{-iky - \pi\theta} \Gamma_a(k, \pi). \quad (38)$$

Thus, we have a one to one map between a function of the deformed superspace and a function of ordinary superspace. Now we can write the product of two deformed superfields as

$$\begin{aligned} \hat{\Gamma}^a(\hat{y}, \hat{\theta}) \hat{\Gamma}_a(\hat{y}, \hat{\theta}) &= \int d^3 k_1 d^3 k_2 \int d^2 \pi_1 d^2 \pi_2 \\ &\times \exp -i((k_1 + k_2)\hat{y} + (\pi_1 + \pi_2)\hat{\theta}) \\ &\times \exp -\frac{i}{2} (A^{\mu a} (\pi_{-a}^2 k_\mu^1 - k_\mu^2 \pi_{-a}^1)) \\ &\times \Gamma^a(k_1, \pi_1) \Gamma_a(k_2, \pi_2). \end{aligned} \quad (39)$$

This motivates the definition of the star product for the functions on ordinary superspace,

$$\begin{aligned} \Gamma^a(y, \theta) \star \Gamma_a(y, \theta) &= \exp -\frac{i}{2} (+A^{\mu a} (\partial_{-a}^2 \partial_\mu^1 - \partial_\mu^2 \partial_{-a}^1)) \\ &\times \Gamma^a(y_1, \theta_1) \Gamma_a(y_2, \theta_2) \big|_{y_1=y_2=y, \theta_1=\theta_2=\theta}. \end{aligned} \quad (40)$$

It is also useful to define the following bracket

$$[\Gamma^a \star \Gamma_a] = T_A f_{BC}^A \Gamma^{aB} \star \Gamma_a^C. \quad (41)$$

In order to construct a Yang-Mills theory on this deformed superspace, it is useful to define

$$\omega_a = \frac{1}{2} D^b D_a \Gamma_b - \frac{i}{2} [\Gamma^b \star D_b \Gamma_a] - \frac{1}{6} [\Gamma^b \star [\Gamma_b \star \Gamma_a]]. \quad (42)$$

The bulk non-abelian Yang-Mills theory on this deformed superspace is given by

$$\mathcal{L}_c = \nabla^2 \text{Tr}(\omega^a \star \omega_a)|. \quad (43)$$

So, the boundary theory can be written as

$$\mathcal{L}_c = \nabla_+ \text{Tr}(\Omega(\theta_+))_{\theta_+=0}, \quad (44)$$

where

$$\Omega = \nabla_- (\omega^a \star \omega_a)_{\theta_-=0}. \quad (45)$$

Thus, we have a undeformed theory on the boundary. This is because the boundary breaks half of the supersymmetry and we have chosen to break the supersymmetry corresponding to Q_- , which is deformed in the bulk.

3 Batalin-Vilkovisky Formalism

Some of the degrees of freedom in the bulk super-Yang-Mills theory are not physical because it is invariant under the following super-gauge transformations

$$\delta \Gamma_a = \nabla_a \star \Lambda, \quad (46)$$

where $\Lambda = \Lambda^A T_A$. This gauge freedom of the bulk theory also induces a gauge freedom for the boundary theory. So, in order to quantise the boundary theory we will have to fix a gauge. This can be done at a quantum level by adding a gauge fixing term and a ghost term to the original classical Lagrangian density. In order to do that we first define a matrix valued scalar superfield B along with two matrix valued anti-commutating superfields c and \bar{c} . These superfields are suitably contracted with generators of the Lie algebra in the adjoint representation,

$$\begin{aligned} F(y, \theta) &= F^A(y, \theta) T_A, & c(y, \theta) &= c^A(y, \theta) T_A, \\ \bar{c}(y, \theta) &= \bar{c}^A(y, \theta) T_A. \end{aligned} \quad (47)$$

Now we can write the gauge fixing term \mathcal{L}_{gf} and the ghost term \mathcal{L}_{gh} as follows:

$$\begin{aligned} \mathcal{L}_{gf} &= \nabla_+ \text{Tr}(\mathcal{N}(\theta_+))_{\theta_+=0}, \\ \mathcal{L}_{gh} &= \nabla_+ \text{Tr}(\mathcal{K}(\theta_+))_{\theta_+=0}, \end{aligned} \quad (48)$$

where

$$\begin{aligned}\mathcal{N} &= \nabla_-(F \star D^a \Gamma_a)_{\theta_-=0}, \\ \mathcal{K} &= \nabla_-(\bar{c} D^a \nabla_a \star c)_{\theta_-=0}.\end{aligned}\tag{49}$$

The total Lagrangian density which is given by the the sum of the original Lagrangian density with the gauge fixing term and the ghost term

$$\mathcal{L} = \mathcal{L}_c + \mathcal{L}_{gf} + \mathcal{L}_{gh}.\tag{50}$$

The BRST transformations are given by

$$\begin{aligned}s\Gamma_a &= \nabla_a \star c, & sc &= -\frac{1}{2}[c \star c], \\ s\bar{c} &= -F, & sF &= 0,\end{aligned}\tag{51}$$

and the anti-BRST transformations are given by

$$\begin{aligned}\bar{s}\Gamma_a &= (D_a - i\Gamma_a) \star \bar{c}, & \bar{s}c &= F - [c \star \bar{c}], \\ \bar{s}\bar{c} &= -\frac{1}{2}[\bar{c} \star \bar{c}], & \bar{s}F &= [F \star \bar{c}],\end{aligned}\tag{52}$$

where

$$[c \star c] = T_A f_{BC}^A c^B \star c^C.\tag{53}$$

Both these transformations are nilpotent, $s^2 = \bar{s}^2 = 0$. In fact, they also satisfy $s\bar{s} + \bar{s}s = 0$. The sum of the gauge fixing term and the ghost term can be expressed as

$$\begin{aligned}\mathcal{L}_{gf} + \mathcal{L}_{gh} &= -\nabla_+ \frac{s\bar{s}}{2} Tr(\mathcal{Z}(\theta_+))_{\theta_+=0} \\ &= \nabla_+ \frac{\bar{s}s}{2} Tr(\mathcal{Z}(\theta_+))_{\theta_+=0}.\end{aligned}\tag{54}$$

where

$$\mathcal{Z} = \nabla_-(\Gamma^a \star \Gamma_a)_{\theta_-=0}.\tag{55}$$

So, we can obtain undeformed gauge fixing and ghost terms for the boundary theory, even though the bulk theory is deformed. Furthermore, the total Lagrangian density is invariant under the BRST and the anti-BRST transformations. This is because the BRST and the anti-BRST transformations are nilpotent and the sum of the gauge fixing term and the ghost term can

be written as a total BRST and a total anti-BRST variation. Hence, any BRST or anti-BRST transformation of the sum of the gauge fixing term and the ghost term vanishes. The BRST or the anti-BRST transformations of the original classical Lagrangian density are ghost or anti-ghost valued gauge transformations and so they also vanish. Hence, the total Lagrangian density is invariant under both the BRST and the anti-BRST transformations, $s\mathcal{L} = \bar{s}\mathcal{L} = 0$. Now we let

$$\begin{aligned}\mathcal{X} &= -\frac{s\bar{s}}{2}\mathcal{Z}(\theta_+) \\ &= \frac{\bar{s}s}{2}\mathcal{Z}(\theta_+).\end{aligned}\tag{56}$$

Now the total Lagrangian density given by Eq. (50), can be written as

$$\mathcal{L}_c = \nabla_+ Tr(\mathcal{X}(\theta_+) + \Omega(\theta_+))_{\theta_+=0}.\tag{57}$$

So, the deformation of the bulk theory does not effect the boundary theory even at a quantum level.

We will analyse this theory in background field method. So, we start by shift all the original fields as

$$\begin{aligned}\Gamma_a &\rightarrow \Gamma_a - \tilde{\Gamma}_a, \\ c &\rightarrow c - \tilde{c}, \\ \bar{c} &\rightarrow \bar{c} - \tilde{\bar{c}}, \\ F &\rightarrow F - \tilde{F}.\end{aligned}\tag{58}$$

Now we require the Lagrangian density to be invariant under both the original BRST transformations and these shift transformations,

$$\tilde{\mathcal{L}} = \mathcal{L}(\Gamma_a - \tilde{\Gamma}_a, c - \tilde{c}, \bar{c} - \tilde{\bar{c}}, F - \tilde{F}).\tag{59}$$

It will be useful to define $\tilde{\nabla}_a$ as

$$\tilde{\nabla}_a = D_a - i\Gamma_a + i\tilde{\Gamma}_a.\tag{60}$$

The extended BRST transformations are given by

$$s\Gamma_a = \psi_a, \quad s\tilde{\Gamma}_a = \psi_a - \tilde{\nabla}_a \star (c - \tilde{c}),\tag{61}$$

$$\begin{aligned}sc = \epsilon &\quad s\tilde{c} = \epsilon + \frac{1}{2}[(c - \tilde{c}) \star (c - \tilde{c})], \\ s\bar{c} = \bar{\epsilon}, &\quad s\tilde{\bar{c}} = \bar{\epsilon} + (F - \tilde{F}), \\ sF = \psi, &\quad s\tilde{F} = \psi,\end{aligned}\tag{62}$$

where

$$\tilde{\nabla}_a \star (c - \tilde{c}) = D_a \star (c - \tilde{c}) - i(\Gamma_a - \tilde{\Gamma}_a) \star (c - \tilde{c}), \quad (63)$$

and $\psi_a, \epsilon, \bar{\epsilon}, \psi$ are the ghost superfields associated with the shift symmetries of the original fields Γ_a, c, \bar{c}, F , respectively. The BRST transformations of these superfields vanish,

$$\begin{aligned} s\psi_a &= 0, & s\epsilon &= 0, \\ s\bar{\epsilon} &= 0, & s\psi &= 0, \end{aligned} \quad (64)$$

Now we define anti-fields $\Gamma_a^*, c^*, \bar{c}^*$ and F^* , such that they have opposite parity to the original fields. The BRST transformations of these anti-fields is given by

$$\begin{aligned} s\Gamma_a^* &= -b_a, & sc^* &= -B, \\ s\bar{c}^* &= -\bar{B}, & sF^* &= -b, \end{aligned} \quad (65)$$

where b_a, B, \bar{B} and b are new auxiliary superfields. The BRST transformations of these new auxiliary superfields vanishes

$$\begin{aligned} sb_a &= 0, & sB &= 0, \\ s\bar{B} &= 0, & sb &= 0. \end{aligned} \quad (66)$$

The Lagrangian density to gauge fix the shift symmetry is chosen to make the tilde fields to vanish, so as to obtain the original Lagrangian density,

$$\tilde{\mathcal{L}}_{gf} + \tilde{\mathcal{L}}_{gh} = \nabla_+ Tr(\mathcal{W}(\theta_+))_{\theta_+=0}, \quad (67)$$

where

$$\begin{aligned} \mathcal{W} &= \nabla_- (-b^a \star \tilde{\Gamma}_a + \Gamma^{*a} \star (\psi_a - \tilde{\nabla}_a \star (c - \tilde{c})) \\ &\quad - \bar{B} \star \tilde{c} + \bar{c}^* \star \left(\epsilon + \frac{1}{2} [(c - \tilde{c}) \star (c - \tilde{c})] \right) \\ &\quad + B \star \tilde{\bar{c}} - c^* \star (\bar{\epsilon} + (F - \tilde{F}) + b \star \tilde{F} \\ &\quad + F^* \star \psi)_{\theta_-=0}. \end{aligned} \quad (68)$$

The tilde fields vanish upon integrating out the auxiliary superfields. This Lagrangian density is invariant under the original BRST transformation and

the shift transformations. We also have the original Lagrangian density, which is only a function of the original fields,

$$\mathcal{L}'_{gf} + \mathcal{L}'_{gh} = \nabla_+ Tr(\mathcal{U}(\theta_+))_{\theta_+=0}, \quad (69)$$

where

$$\mathcal{U} = \nabla_- \left(-\frac{\delta\Psi}{\delta\Gamma_a} \star \psi_a + \frac{\delta\Psi}{\delta c} \star \epsilon + \frac{\delta\Psi}{\delta\bar{c}} \star \bar{\epsilon} - \frac{\delta\Psi}{\delta F} \star \psi \right)_{\theta_-=0}. \quad (70)$$

The classical Lagrangian density can now be written as

$$\mathcal{L}_c = \nabla_+ Tr(\bar{\Omega}(\theta_+))_{\theta_+=0}, \quad (71)$$

where

$$\bar{\Omega} = \nabla_- ((\omega^a - \tilde{\omega}^a) \star (\omega_a - \tilde{\omega}_a))_{\theta_-=0}. \quad (72)$$

Now the total Lagrangian density is given by

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_c + \tilde{\mathcal{L}}_{gf} + \tilde{\mathcal{L}}_{gh} + \mathcal{L}'_{gf} + \mathcal{L}'_{gh} \\ &= \nabla_+ Tr(\bar{\Omega}(\theta_+) + \mathcal{U}(\theta_+) + \mathcal{W}(\theta_+))_{\theta_+=0}. \end{aligned} \quad (73)$$

If we integrate out the fields, setting the tilde fields to zero, we have

$$\mathcal{L} = \nabla_+ Tr(\mathcal{M}(\theta_+))_{\theta_+=0}, \quad (74)$$

where

$$\begin{aligned} \mathcal{M} &= \nabla_- \left(\Omega + \Gamma^{*a} \star \nabla_a c + \frac{1}{2} \bar{c}^* \star [c \star c] - c^* \star F \right. \\ &\quad - \left(\Gamma^{*a} + \frac{\delta\Psi}{\delta\Gamma_a} \right) \star \psi^a + \left(\bar{c}^* + \frac{\delta\Psi}{\delta c} \right) \star \epsilon - \left(c^* + \frac{\delta\Psi}{\delta\bar{c}} \right) \star \bar{\epsilon} \\ &\quad \left. + \left(F^* - \frac{\delta\Psi}{\delta F} \right) \star \psi \right)_{\theta_-=0}. \end{aligned} \quad (75)$$

Thus, even after shifting all the fields we obtained a undeformed boundary theory, even though our bulk theory was deformed. This result even holds at a quantum level. By integrating out the ghosts associated with the shift symmetry, we obtain explicit expression for the anti-fields,

$$\begin{aligned} \Gamma^{*a} &= -\frac{\delta\Psi}{\delta\Gamma_a}, & \bar{c}^* &= -\frac{\delta\Psi}{\delta c}, \\ c^* &= \frac{\delta\Psi}{\delta\bar{c}}, & F^* &= \frac{\delta\Psi}{\delta F}. \end{aligned} \quad (76)$$

With these identifications we obtain an explicit form for the total Lagrangian density in background field method.

This Lagrangian density is invariant under the extended BRST transformations. However, the original Lagrangian density was also invariant under the anti-BRST transformations. So, this Lagrangian density, must also be invariant under on-shell extended anti-BRST transformation as the on-shell extended anti-BRST transformations reduce to the original anti-BRST transformations [16]. The extended anti-BRST transformation of the original superfields is given by

$$\begin{aligned}\bar{s}\Gamma_a &= \Gamma_a^* + \tilde{\nabla}_a \star (c - \tilde{c}), \\ \bar{s}c &= c^* + (F - \bar{F}) - [(c - \tilde{c}) \star (\bar{c} - \tilde{\bar{c}})], \\ \bar{s}\bar{c} &= \bar{c}^* - \frac{1}{2}[(\bar{c} - \tilde{\bar{c}}) \star (\bar{c} - \tilde{\bar{c}})], \\ \bar{s}F &= F^* + [(F - \tilde{F}) \star (\bar{c} - \tilde{\bar{c}})],\end{aligned}\tag{77}$$

and the extended anti-BRST transformations of the shifted superfields is given by

$$\begin{aligned}\bar{s}\tilde{\Gamma}_a &= \Gamma_a^*, & \bar{s}\tilde{c} &= c^*, \\ \bar{s}\tilde{\bar{c}} &= \bar{c}^*, & \bar{s}\tilde{F} &= F^*.\end{aligned}\tag{78}$$

The anti-BRST transformations of the ghost superfields is given by,

$$\begin{aligned}\bar{s}\psi_a &= b_a + \tilde{\nabla}_a \star (F - \tilde{F}) - [(\tilde{\nabla}_a \star (c - \tilde{c})) \star (\bar{c} - \tilde{\bar{c}})], \\ \bar{s}\epsilon &= B - [(F - \tilde{F}) \star (\bar{c} - \tilde{\bar{c}})] + [[(\bar{c} - \tilde{\bar{c}}) \star (c - \tilde{c})] \star (c - \tilde{c})], \\ \bar{s}\bar{\epsilon} &= \bar{B} - [(F - \tilde{F}) \star (\bar{c} - \tilde{\bar{c}})], \\ \bar{s}\psi &= b,\end{aligned}\tag{79}$$

The extended anti-BRST transformations of the anti-fields and the new auxiliary fields vanishes,

$$\begin{aligned}\bar{s}b_a &= 0, & \bar{s}\Gamma_a^* &= 0, \\ \bar{s}B &= 0, & \bar{s}c^* &= 0, \\ \bar{s}\bar{B} &= 0, & \bar{s}\bar{c}^* &= 0, \\ \bar{s}\bar{b} &= 0, & \bar{s}F^* &= 0.\end{aligned}\tag{80}$$

So, we obtained an undeformed classical Lagrangian density, gauge fixing term and the ghost term for the boundary theory, even though the bulk theory was deformed. Furthermore, these results also hold at a quantum level.

3.1 Conclusion

In this paper we analysed the Yang-Mills theory in a deformed superspace, where the deformation was caused by noncommutativity between bosonic and fermionic coordinates. We deformed the bulk theory in such a way that we could obtain an undeformed theory on the boundary. We analysed the BRST and the anti-BRST symmetries of this boundary theory in the BV-formalism. The ghost and gauge fixing terms for the boundary theory were not effected by deformations in the bulk. This implied that this result also hold at a quantum level.

The spacelike noncommutative field theories are known to be unitarity [36]-[37]. However, due to Eq. (40), higher order temporal derivatives will occur in the product of fields due to spacetime noncommutativity. It is well known that the evolution of the S -matrix is not unitary for the field theories with higher order temporal derivatives [38]-[41]. Thus, spacetime noncommutativity will break the unitarity of the resultant theory. However, if we restrict the theory to spacelike noncommutativity and thus do not include any higher order temporal derivatives, then this problem can be avoided. In fact, in the case of spacelike noncommutativity it is possible to construct the Northey's charges [42]-[43]. Thus, if we restrict the spacetime deformations to spacelike noncommutativity, then we can construct the Northey's charges for this deformed theory. It will be interesting to construct the BRST and the anti-BRST charges for this theory and use them to find the physical states of this theory.

References

- [1] D. V. Belyaev and P. van Nieuwenhuizen, JHEP. 0804, 008 (2008)
- [2] D. V. Belyaev, JHEP. 0601, 046 (2006)
- [3] D. V. Belyaev, JHEP 0601, 047 (2006)
- [4] S. Ferrara and E. Sokatchev, Phys. Lett. B579, 226 (2004)
- [5] P. A. Grassi and R. Ricci and D. R. Llana, JHEP. 0407, 065 (2004)
- [6] K. Ito, Y. Kobayashi and S. Sasaki, JHEP. 0704, 011 (2007)
- [7] K. Ito and S. Sasaki, JHEP, 0611, 004 (2006)

- [8] I. Adam and Y. Oz, JHEP. 0509, 002 (2005)
- [9] J. Fjelstad and S. Hwang, Phys. Lett. B466, 227 (1999)
- [10] Izumi Ojima. Prog. Theor. Phys., 64:625, 1980.
- [11] N. Nakanishi and I. Ojima, Covariant operator formalism of gauge theories and quantum gravity. World. Sci. Lect. Notes. Phys. (1990)
- [12] M. Chaichian and W. F. Chen, Z.Y. Zhu, Phys. Lett. B387, 785 (1996)
- [13] L.P. Colatto, M.A. De Andrade, O.M. Del Cima, D.H.T. Franco, J.A. Helayel-Neto and O. Piguet, J. Phys. G24, 1301 (1998)
- [14] C. P. Constantinidis, O. Piguet and W. Spalenza, Eur. Phys. J. C33, 443 (2004)
- [15] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. B 102, 27 (1981)
- [16] M. Faizal and M. Khan, Eur. Phys. J. C 71, 1603 (2011)
- [17] P. Gregoire and M. Henneaux, J. Phys. A26, 6073 (1993)
- [18] P. Gregoire, M. Henneaux, Commun. Math. Phys. 157, 279 (1993)
- [19] P. Gregoire, M. Henneaux, Phys. Lett. B277, 459 (1992)
- [20] I. A. Batalin and G. A. Vilkovisky, Phys. Rev. D 28, 2567 (1983)
- [21] C. Bizdadea and S. O. Saliu, J. Phys. A 31, 8805 (1998)
- [22] C. Bizdadea, I. Negru and S. O. Saliu, Int. J. Mod. Phys. A 14, 359 (1999)
- [23] J. Qiu and M. Zabzine, Commun. Math. Phys. 300, 789 (2010)
- [24] W. G. Ney, O. Piguet and W. Spalenza, Eur. Phys. J. C36, 245 (2004)
- [25] N. Ikeda, JHEP. 0210, 076 (2002)
- [26] C. I. Lazaroiu and R. Roiban, JHEP. 0203, 022 (2002)
- [27] G. Barnich, R.Constantinescu and P. Gregoire, Phys. Lett. B 293, 353 (1992)

- [28] H. Ikemori, Mod. Phys. Lett. A 7, 3397 (1992)
- [29] N. R. F. Braga and A. Das, Nucl. Phys. B 42, 655 (1995)
- [30] N. R. F. Braga and S. M. de Souza, Phys. Rev. D 53, 916 (1996)
- [31] M. Faizal, JHEP. 1204, 017 (2012)
- [32] M. Faizal and D. J . Smith, Phys. Rev. D85, 105007 (2012)
- [33] M. Faizal, Phys. Rev. D 84, 106011 (2011)
- [34] M. Faizal, Europhys. Lett. 98, 31003 (2012)
- [35] S. J. Gates Jr, M. T. Grisaru , M. Rocek and W. Siegel, Front. Phys. 58, 1 (1983)
- [36] J. Gomis and T. Mehen, Nucl. Phys. B 591, 265 (2000)
- [37] N. Seiberg, L. Susskind and N. Toumbas, JHEP. 0006, 044 (2000)
- [38] K. S. Stelle, Phys. Rev. D 16, 953 (1977)
- [39] J. Julve and M. Tonin, Nuovo Cim. B 46, 137 (1978)
- [40] A. Salam and J. Strathdee, Phys. Rev. D 18, 4480 (1978)
- [41] I.G. Avramidi and A.D. Barvinsky, Phys. Lett. B 159, 269 (1985)
- [42] A. Micu and M. M. S. Jabbari, JHEP. 0101, 025 (2001)
- [43] M. Soroush, Phys. Rev. D 67, 105005 (2003)